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SUB-THRESHOLD STUDY OF THE AZIMUTHAL MODES OF THE KADOMTSEV PLASMA INSTABILITY

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SUB-THRESHOLD STUDY OF THE AZIMUTHAL MODES OF THE KADOMTSEV PLASMA INSTABILITY*

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Anomalous effects appear when a positive column plasma is immersed in a strong longitudinal dc magnetic field (ref. 1). These effects were, to a large extent, explained by the stability considerations presented by Kadomtsev and Nedospasov (ref. 2) who linearized the momentum and continuity equations for electrons and ions and showed that a helical perturbation will grow exponentially when the longitudinal magnetic field exceeds a certain critical value, B_c.

In discussing the anomalous state of the positive column it is common practice (refs. 3-5) to use the linearized theory to predict the critical value of the magnetic field (B_c) and the characteristics of the fully developed helical instability. However, non-linear effects are essential to the attainment of the equilibrium anomalous condition which is normally studied experimentally. In this connection Ichimaru (ref. 6) recently pointed out that a sub-threshold study, that is when $B < B_c$, should yield valuable information about the onset of the instability. In particular the experimental study of the plasma when it is perturbed at frequencies in the neighborhood of the helix frequency and when the magnetic field is brought close to, but does not reach, the critical value B_c, will more closely match the assumptions of the linear perturbation theory.

Excitation of a helical perturbation in a magnetoplasma was first reported by Nishida, et al (ref. 7) in a mercury vapor discharge at a pressure of several millitorr. In the following we report the extension of the technique to different experimental conditions, and its application to the study of the helical modes of the Kadomtsev instability.

We have performed a sub-threshold experiment on a helium positive column in a longitudinal magnetic field at a pressure of 0.38 Torr. A variable frequency signal source was connected between a pair of probes inserted in the discharge and spaced 180° apart in a cross-sectional plane. A second similar pair of probes, located 40 cm farther down the column was used to detect the transmitted signal. In the absence of a transmitted signal strong oscillations, indicating the onset of a Kadomtsev type instability, occurred at a magnetic field strength of 2100 gauss.

The frequency spectrum of the received signal as a function of the applied magnetic field is shown in Figure 1. As the critical field strength was approached a distinct resonance occurred in the received signal. The plot of signal amplitude versus frequency obtained by Nishida et al. showed a broad maximum.

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By contrast, the well developed resonance effect which we obtained enables quantitative comparisons to be made between the linear and non-linear regimes of plasma behavior. Thus the sub-threshold resonance frequency shown in Figure 1 is 16.2 kc whilst with $B = B_c$ the plasma exhibited self sustaining oscillations at a frequency of 16.3 kc. Clearly, in this case, the use of the linear assumption in the Kadomtsev theory does not of itself introduce substantial error in the calculation of the frequency of the non-linear strong oscillation.

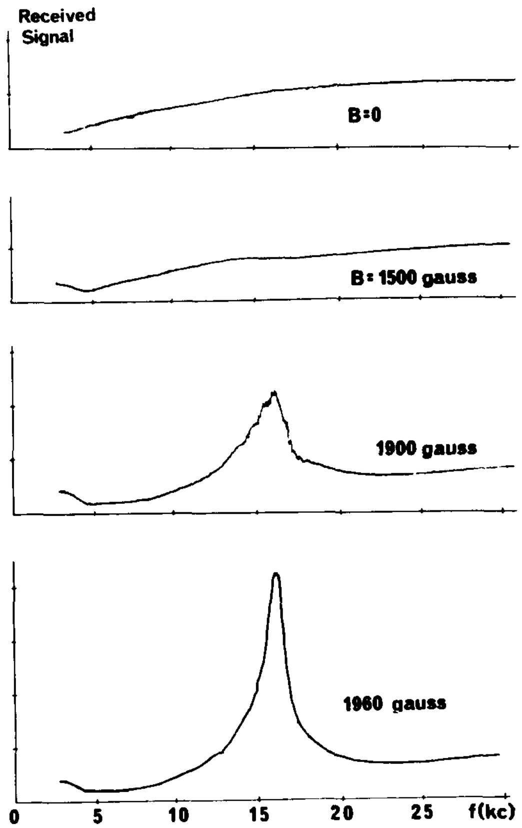

Figure 1. Sub-Threshold Response Spectra of a Helium Positive Column at a Pressure of 0.38 Torr. B_c = 2100 gauss and f_c = 16.3 kc.

The second objective of this letter is to show how this technique may be applied to the study of the several modes of the Kadomtsev instability. We have previously shown both theoretically and experimentally how these modes are affected by the presence of a weak transverse magnetic field (ref. 8). The theory predicts that the $m = 2$ mode becomes less stable than the $m = 1$ mode when the transverse magnetic field exceeds a certain value. Experimental data cited in the previous study showed a fully developed oscillation of the $m = 1$ type occurring in the presence of small transverse magnetic fields and a fully developed oscillation of the $m = 2$ type occurring for larger transverse fields. The possibility existed that the $m = 1$ mode was basic to all these instabilities and was initially present when the critical conditions were met but was then overtaken in the case of larger transverse fields by the $m = 2$ mode as the non-linear mechanisms began to control the growth of the instability. If this were the case the $m = 2$ mode would require the initial presence of the $m = 1$ mode, reminiscent of the observation that the $m = 0$ mode requires the $m = 1$ mode to drive it, as we have previously shown (ref. 8).

The sub-threshold excitation technique may be simply extended to resolve this question, as we show in the results presented in Figure 2. Sub-threshold response spectra are shown over an extended frequency range for two values of the transverse magnetic field. The frequency of the oscillation of the fully developed instability is indicated by an arrow in each case.

The upper curve shows a plasma condition for which the $m = 1$ mode is less stable (or grows more rapidly) than the $m = 2$ mode. Under this condition the $m = 1$ mode dominated the fully developed instability. By contrast the lower curve illustrates a plasma condition for which the $m = 2$ mode is less stable than the $m = 1$ mode, and in the same way it is now the $m = 2$ mode which grows to dominate the fully developed instability. Thus the $m = 2$ mode is a genuinely independent mode of instability.

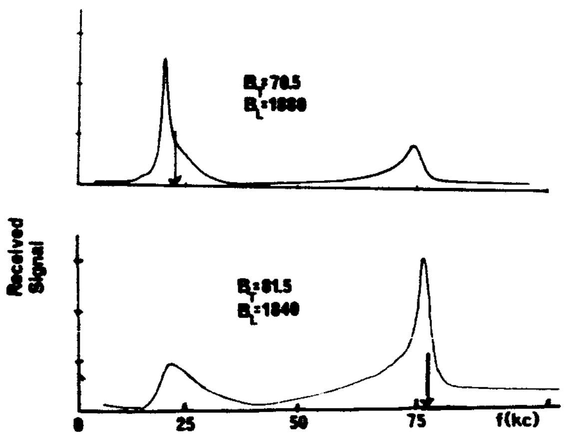

Figure 2. Sub-Threshold Response Spectra as a Function of the Applied Transverse Magnetic Field. The arrows indicate the frequency of self sustaining oscillations that appear when the magnetic field is increased above the critical value (B_c).

We conclude that a sub-threshold study can yield valuable information about plasma instabilities. In the present case, our pre-onset study has provided experimental confirmation of the theoretical prediction that a switch from the $m = 1$ mode to the $m = 2$ mode will occur when a suitable increase is made in the strength of the transverse magnetic field applied to a positive column plasma immersed in a strong longitudinal magnetic field.

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